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Assignment No: 1
SIG 787 - Mathematics for AI Trimester 1, 2025

Index

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Question (1)

(1) The function:

$$f(x) = \begin{cases} \sqrt{x^2 - 2x + 1} & |x| \le c \\ ax^2 + bx + 2 & |x| > c \end{cases}$$

is defined for some $c \in \mathbb{N}$. Answer the following questions.

- Set c as the most right non-zero in your Student ID.
- find the equivalent form of this function as a piecewise defined function by removing absolute value and square root signs.

Solution :-

$$f(x) = \begin{cases} \sqrt{x^2 - 2x + 1} & |x| \le c \\ ax^2 + bx + 2 & |x| > c \end{cases}$$

hint given: Set c as the most right non-zero digit in your Student ID

(1) ∴ Determining C
∴ Student ID : 225440315
∴ Rightmost non-zero digit 5
∴ So $c = 5$

(2) It is define for some $c \in N$ here $c = 5$ hence our Condition is Satisfied.

Step : 1 Understanding the interval $|x| \leq 5$

here inequality $|x| \leq 5$ is an Absolute value inequality, which describes all (R) Real value number x whose distance from 0 on the number line is less than or equal to 5

Definition of Absolute value:

The absolute value $|x|$ represents the non-negative distance of x from 0, mathematically

$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

Breaking down $|x| \leq 5$

The inequality $|x| \leq 5$ means

- The distance of x from 0 is no more than 5.
- This Includes all numbers between -5 and 5 (including -5 and 5 them selves.

Solving $|x| \leq 5$

To remove the absolute value, we consider the two possible cases based on the definition $|x|$:

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Case 1: $x \geq 0$

$|x| = x \leq 5 \Rightarrow 0 \leq x \leq 5$

Case 2 $x < 0$

$|x| = -x \leq 5 \Rightarrow x \geq -5$

multiplying both sides by -1 reverse inequality.

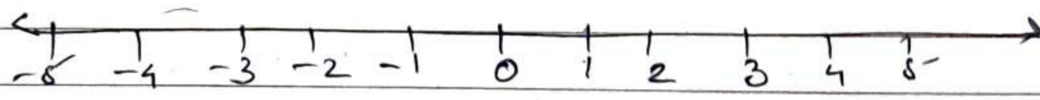
Combining Both cases.

- from case 1: x is between ~~a~~ and 5 (inclusive)
- from case 2: x is between -5 and 0 (since $x<0$)

Thus combining both we get

$$-5 \leq x \leq 5$$

Visual Representation on the Number lines.



- All numbers from -5 to 5 (including endpoints) satisfy $|x| \leq 5$

Conclusion:

The absolute value inequality $|x| \leq c$ where $c>0$) always translates to

$$-c \leq x \leq c$$

This is standard rule for solving such inequality

General Rule for $|x| \leq c$ for any $c > 0$:

$$|x| \leq c \Rightarrow -c \leq x \leq c$$

Understanding the interval $|x| > c$

The inequality $|x| > c$ describes all real number x whose distance from 0 is strictly greater than c.

1. Intuitive explaination:-

- The absolute value (x) measures distance from 0 on the number line
- if $|x| > 0$, then x must be farther than c units away from 0 in either direction (left or right)

- This means:-
  - Number larger than c: $x > 0$
  - number Smaller than $-c$: $x < -c$

2. Mathematical Breakdown

The inequality $|x| > c$ (where $c \geq 0$) Split into 2 cases.

1. Case 1: $x > c$
   Example if $c = 5$, then $x = 6$ gives $|6|$
   $6 > 5$

2. Case 2: gives $x < -c$
   Example if $c = 5$, then $x = -6$ gives $|-6| = 6 > 5$

Thus solution is

$$x < -c \text{ or } x > c$$

Step 2 : Simplifying the function

Now for $|x| \le 5$ (ie $-5 \le x \le 5$) interval the function is

$\sqrt{x^2 - 2x + 1}$, Simplifying equation within Square root.

so, the squareroot becomes

$a^2 - 2ab + b^2 = (a-b)^2$

$$\sqrt{(x-1)^2} = |x-1|$$

hence $x^2 - 2x + 1 = (x-1)^2$

Thus function can be written as

$$f(x) = \begin{cases} |(x-1)| & \text{if } -5 \le x \le 5 \\ ax^2 + bx + 2 & \text{if } x < -5 \text{ or } x > 5 \end{cases}$$

Removing the absolute value.
the expression $|x-1|$ can be split based on the value of x

if $x - 1 \geq 0$ (i.e $x \geq 1$) then $|x-1| = \boxed{x-1}$

if $x - 1 < 0$ (i.e. $x < 1$) then $-|x-1| = -(x-1) = 1 - x$

hence with interval $-5 \le x \le 5$ the split occurs at $x = 1$
therefor function becomes

$$f(x) = \begin{cases} (1-x) & \text{if } -5 \le x < 1 \\ (x-1) & \text{if } 1 \le x \le 5 \\ ax^2 + bx + 2 & \text{if } x < -5 \text{ or } x > 5 \end{cases}$$

Question (2)

find the derivative of $f(x)$ and determine where the function is not differentiable

find the derivative $f'(x)$ for all the interval.

1. for interval $-5 \leq x < 1$ $f(x)$ is $(1-x)$.

$$f(x) = (1-x) \quad \text{if } -5 \leq x < 1$$

$$\boxed{f'(x) = -1}$$

2. for Interval $1 \leq x \leq 5$ $f(x)$ is $(x-1)$

$$\boxed{f'(x) = 1}$$

3. for interval $x < -5$ or $x > 5$

$$f(x) = ax^2 + bx + 2$$

$$\boxed{f'(x) = 2ax + b}$$

At $x = 1$

The function changes from $(1-x)$ to $(x-1)$
lets check the derivative at $x=1$

- left derivative $(x \to 1^-)$

$$f'(1^-) = -1$$

- Right derivative $(x \to 1^+)$

$$f'(1^+) = 1$$

Since $f'(1^-) \neq f'(1^+)$, the function is not - non - differentiable at $x = 1$

So now $x = -5$ and $x = 5$:
we need to check the differentiability at the boundaries where the function changes its definition

At $x = 5$

- left limit $(x \to 5^-)$:

$$f(5^-) = (x-1) = 5-1 = 4$$
$$f(5^-) = 4$$
$$f'(5^-) = 1 \quad \therefore \text{as } f'(x) = (x-1)' = 1$$

- Right hand limit $(x \to 5^+)$

$$f(5^+) = a(5)^2 + b(5) + 2$$
$$= 25a + 5b + 2$$
$$f'(5^+) = 10a + b$$

for continuity at $x = 5$

$$25a + 5b + 2 = 4$$
$$\therefore 25a + 5b = 2 \quad \text{— Equation (1)}$$

for differentiability at $x = 5$

$$\therefore 10a + b = 1 \quad \text{— Equation (2)}$$

Now Solving Equation 1 and 2
from equation 2

$$b = 1 - 10a$$

Substitute into equation 1

$$25a + 5(1-10a) = 2$$
$$25a + 5 - 50a = 2$$
$$-25a = -3$$
$$a = \frac{3}{25}$$

Then

$$b = 1 - 10(3/5) = 1 - 2(3/5) = -1/5$$

$$\boxed{\therefore \; a = 3/25 \quad \text{when } x = 5 \qquad b = -1/5}$$

At $x = -5$

- left hand limit $(x \to -5^-)$

$$f(-5^-) = a(-5)^2 + b(-5) + 2$$
$$= 25a - 5b + 2$$
$$f'(-5^-) = 2a(-5) + b = -10a + b$$
$$= -10a + b$$

- Right side limit. $(x \to -5^+)$

$$f(-5^+) = 1 - (-5) = 6$$
$$f'(-5^+) = -1$$

for continuity at $x = -5$

$$25a - 5b + 2 = 6$$
$$25a - 5b = 4 \text{ (equation 3)}$$

for differentiability at $x = -5$

$$-10a + b = -1 \quad \text{(equation 4)}$$

Now solving equation 3 and 4 from equation 4.

$$b = 10a - 1$$

Substituting into equation 3.

$$25a - 5(10a - 1) = 4$$
$$25a - 50a + 5 = 4$$
$$-25a = -1$$
$$a = 1/25$$

Then

$$b = 10(1/25) - 1 = \frac{10}{25} - 1 = \frac{-15}{25} = -3/5$$

However, we have two sets of values for a and b.

1. from $x = 5$: $a = 3/25$, $b = -1/5$

2. from $x = -5$, $a = 1/25$, $b = -3/5$

[This inconsistancy means that the function cannot be differentiable at both $x = -5$ and $x = 5$ simultaneously. There for we must choose condition to prioritize Typically we ensure differentiability at one boundary and accept non-differentiability at the other.]

Determining where the function is not differentiable.

From the analysis.

1. At $x=1$
   - The left and right derivatives are -1 and 1 respectively.
   - Not differentiable here.

2. At $x=-5$
   - The derivatives from the left and right do not match unless specific conditions are met.
   - with the given a and b : it is not differentiable here.

3. At $x=5$
   - If we choose $a = 3/25$ and $b = -1/5$, the function is differentiable here.
   - however, this causes non-differentiability at $x = -5$

Thus, the function is not differentiable at $x=-5$ and $x=1$, It is differentiable ~~at $x=-5$ and $x=1$~~ every where else including at $x=5$ if we choose corresponding a & b

Final Answer:

1. Simplified functions

$$f(x) = \begin{cases} 1-x & \text{if } -5 \le x < 1 \\ x-1 & \text{if } 1 \le x \le 5 \\ ax^2 + bx + 2 & \text{if } x < -5 \text{ or } x > 5 \end{cases}$$

2. Derivative

$$f'(x) = \begin{cases} -1 & \text{if } -5 \le x < 1 \\ 1 & \text{if } 1 \le x \le 5 \\ 2ax + b & \text{if } x < -5 \text{ or } x > 5 \end{cases}$$

- At $x=1$ $f'(x)$ doesn't exist.
- At $x=-5$ $f'(x)$ doesn't exist unless specific a and b are choosen which would then cause non-differentiabiliy at $x=5$

3. Points of non-differentiabiliy
- The function is not differentiable at $x=-5$ and $x=1$
- It is differentiable everywhere else if we choose $a = 3/25$ and $b = -1/5$ (making it differentiable at $x=5$ but not at $x=-5$

Conclusion:

The function $f(x)$ is not differentiable at $x=-5$ and $x=1$. The exact value of a & b depend on which boundy point we prioritize for differentiabiliy since the same (a,b) pair cannot satisfy both boundary : function is not differentiable at both $x=5$ & $x=-5$

for question 3-9, Consider $a = \frac{1}{9}$, and $b = -\frac{1}{3}$.

Question 3) find all the intercepts of $y = f(x)$

Solution: finding all intercepts of $y = f(x)$

we have solved from previous:

$$f(x) = \begin{cases} 1-x & -5 \le x < 1 \\ x-1 & 1 \le x \le 5 \\ ax^2+bx+2 & x<-5 \text{ or } x>5 \end{cases}$$

Considering $a = \frac{1}{9}$ and $b = \frac{-1}{3}$

$$f(x) = \begin{cases} 1-x & -5 \le x < 1 \\ x-1 & 1 \le x \le 5 \\ \frac{1}{9}x^2 - \frac{1}{3}x + 2 & x<-5 \text{ or } x>5 \end{cases}$$

1. y-intercept (where $x = 0$)

- Since $0 \in [-5, 1)$, we use $f(x) = 1-x$.

$$f(0) = 1-x = 1-0 = 1$$

$\boxed{y \text{ intercept} : (0, 1)}$

2. x- Intercepts (where $y=0$)

we Solve $f(x) = 0$ for each piece.

- for $-5 \le x < 1$:

$$1 - x = 0$$

$x = 1$ which is not in range of $-5 \le x < 1$

$\boxed{\text{No solution here.}}$

- for $1 \le x \le 5$

$$x - 1 = 0$$
$$x = 1$$

$\boxed{\therefore x\text{- intercept } (1, 0)}$

- for $x < -5$ or $x > 5$

$$\frac{1}{9}x^2 - \frac{1}{3}x + 2 = 0$$

$$x^2 - 3x + 18 = 0$$

Discriminating

$$D = (-3)^2 - 4(1)(18) = 9 - 72 = -63 < 0$$

$\boxed{\text{no real roots}}$

Summary of Intercepts

- y - intercept : $(0, 1)$
- x - intercept : $(1, 0)$

Question:

(4) find all critical points of $y = f(x)$.

Solution:

finding all Critical points of $y = f(x)$

critical points occurs where $f'(x) = 0$

or $f'(x)$ = doesn't exists.

1. for $-5 \leq x < 1$

$$f(x) = 1 - x$$

$$\therefore f'(x) = -1 \quad (\text{Never zero})$$

- No critical points here.

2. For $1 \leq x \leq 5$

$$f(x) = x - 1$$

$$\therefore f'(x) = 1 \quad (\text{Never zero})$$

- No critical points here.

3. for $x < -5$ or $x > 5$:

$$f(x) = \frac{1}{9}x^2 - \frac{1}{3}x \cancel{+} 2 \quad - \quad f'(x) = \frac{2}{9}x - \frac{1}{3}$$

Set $f'(x) = 0$:

$$\frac{2}{9}x - \frac{1}{3} = 0 \quad x = 3/2$$

but $\frac{3}{2}$ is not in $x < -5$ or $x > 5$.

No Critical points here.

Boundary points $(x = -5, 1, 5)$

- At $x = -5$:

left derivative $(x \to -5^-)$: $f'(x) = \frac{2}{9}x - \frac{1}{3}$

$\Rightarrow f'(-5) = \frac{-10}{9} - \frac{1}{3} = \frac{-13}{9}$

Righ hand derivative $(x \to -5^+)$: $f'(x) = -1$

- Differentiability fails: $\frac{-13}{9} \neq -1$

critical points at $x = -5$.

- At $x = 1$
- Left derivative $(x \to 1^-)$: $f'(x) = -1$
- Right derivative $(x \to 1^+)$: $f'(x) = 1$.
- Differentiability fails $-1 \neq 1$ critical point

at $x = 1$ Critical points

- At $x = 5$:
- left derivative $(x \to 5^-)$: $f'(x) = 1$.
- Right derivative $(x \to 5^+)$: $f'(x) = \frac{2}{9}(5) - \frac{1}{3} = 7/9$
- Differentiability fails $7/9 \neq 1$

critical point at $x = 5$

final Answers

(3) Intercepts

y - intercepts : $(0, 1)$

x - intercepts : $(1, 0)$.

- Critical points:

    - $\boxed{x = -5}$ $\boxed{x = 1}$ and $\boxed{x = 5}$

note: These critical points arise from corners (where the derivative does not exist), not from $f'(x) = 0$.

Question:

(5) Classify all critical points of $y = f(x)$

To classify the critical points $x = -5$, $x = 1$ and $x = 5$, we analyze the behaviour of the derivative $f'(x)$ arround these points since these are points where the function is not differentiable. we focus on the first derivative test for corners (i.e. points where the left and right derivatives disagree).

1. Critical point at $x = -5$

- left Derivative $(x \to -5^-)$:

$$f'(x) = \frac{2}{9}x - \frac{1}{3} \Rightarrow f'(-5^-) = -\frac{13}{9}$$

- Right Derivative $(x \to -5^+)$:

$$f'(x) = -1$$

Behaviour.
- left Slope ($-13/9 \approx -1.44$) is steeper and negative.
- Right Slope (-1) is less steep and negative.

Conclusion:
The function transitions from a steeper decrease to a gentler decrease. This is not a local extremum but a corner (non-differential point).

Classification

$\boxed{\text{Corner (no extremum).}}$

2. Critical point at $x = 1$
- left derivative ($x \to 1^-$): $f'(x) = -1$.
- Right derivative ($x \to 1^+$): $f'(x) = 1$

Behaviour
- left slope (-1): Decreasing.
- Right slope (1): increasing

Conclusion
The function changes from decreasing to increasing at $x = 1$, indicating a local minimum

Classification.

Local minimum.

3. Critical point at $x = 5$.
- left Derivative ($x \to 5^-$) : $f'(x) = 1$.
- Right Derivative ($x \to 5^+$) ; $f'(x) = 7/9$.

Behaviour:
- left slope (1): increasing
- Right slope ($7/9 \approx 0.78$): increasing but less steep.

Conclusion:
the function remains increasing but with a discontinuity in the derivative. This is a corner (non-differentiable point), not an extremum.

$\boxed{\text{Classification. Corner (no extremum).}}$

Summary of Classifications :-

Critical Point	left Derivative	Right Derivative	Classification
$x = -5$	$-13/9$	-1	corner
$x = 1$	-1	1	local minimum
$x = 5$	1	$7/9$	corner (no extremum)

Key observations:-

1. Only $x=1$ is a local minimum because the function changes from decreasing to increasing there.
2. $x=-5$ and $x=5$ are corners where the derivative changes abruptly but no extrema occurs.

final Answer:-

- $x=-5$: Corner (no extremum)
- $x=1$: Local minimum
- $x=5$: Corner (no extremum).

Question:

(6) find intervals that the function is increasing or decreasing.

we analyze the first derivative $f'(x)$ in each interval.

1. for $x<-5$: $\quad f'(x)=\frac{2}{9}x-\frac{1}{3}$

- At $x=-6$: $f'(-6)=\frac{-12}{9}-\frac{1}{3}=\frac{-15}{9}$

$=\frac{-5}{3}<0$

- Decreasing on $(-\infty,-5)$.

2. for $-5 \le x < 1$:

$$f'(x) = -1 < 0$$

$\boxed{\circ \text{ Decreasing on } [-5, 1).}$

3. for $1 \le x \le 5$:

$$f'(x) = 1 > 0$$

$\boxed{\circ \text{ increasing on } [1, 5].}$

4. for $x > 5$:

$$f'(x) = \frac{2}{9}x - \frac{1}{3}$$

○ At $x = 6$: $f'(6) = \frac{12}{9} - \frac{1}{3} = \frac{9}{9} = 1 > 0$

$\boxed{\circ \text{ increasing on } (5, \infty).}$

Summary:

- Decreasing : $(-\infty, 1)$.
- Increasing : $[1, \infty)$.

Question

(7) find the second derivative of $y = f(x)$

1. for $x < -5$ or $x > 5$

$$f'(x) = \frac{2}{9}x - \frac{1}{3} \Rightarrow f''(x) = \frac{2}{9} > 0$$

- Concave up on $(-\infty, 5) \cup (5, \infty)$.

2. for $-5 \leq x < 1$:

$$f'(x) = -1 \Rightarrow f''(x) = 0$$

- linear (no concavity). ($f'(x)$ is not changing

3. for $1 \leq x \leq 5$;

$$f'(x) = 1 \Rightarrow f''(x) = 0$$

- linear (no concativity). ($f'(x)$ is not changing)

Summary

$$\boxed{f''(x) = \begin{cases} 0 & -5 \leq x \leq 5 \\ 2/9 & x < -5 \text{ or } x > 5 \end{cases}}$$

Question

(8) Points of Inflection of $y = f(x)$

Points of Inflection:
points of Inflection occurs where $f''(x)$ changes sign. Here:

- $f''(x)$ changes from $2/9$ (concave up) to 0 (linear) at $x = -5$
- $f''(x)$ changes from 0 (linear) to $2/9$ (concave up) at $x = 5$.

However, no true inflection points exist because.

- The concavity does not change from "up" to "down" or vice versa, it transitions from "up" to "linear" to "up" again.
- The linear segments $(-5 \le x \le 5)$, have no curvature.

Conclusion:

no points of inflection.

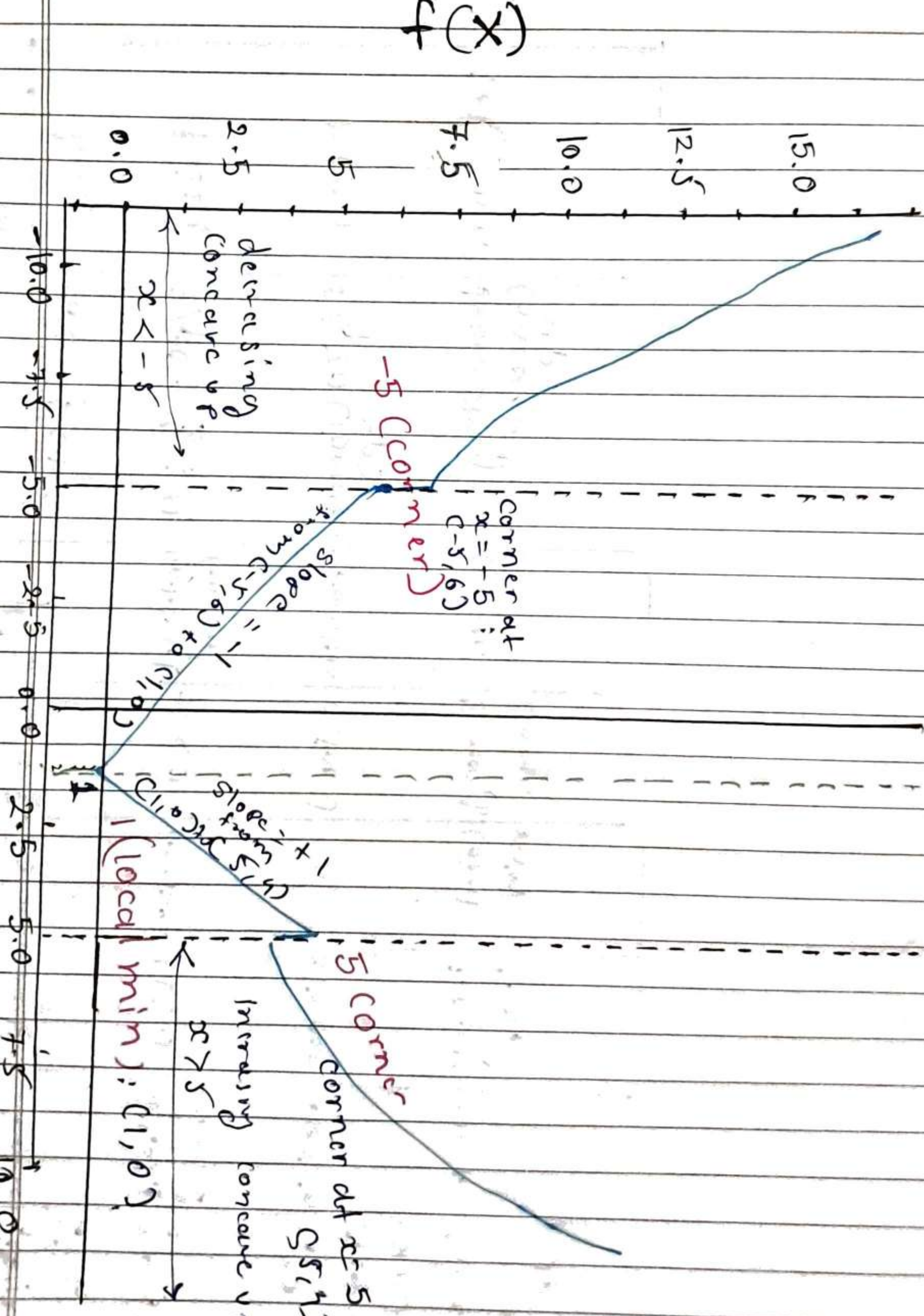
(9) Draw the nction by hand using all the informa-
tion you obtained so far. Annotate important
information on your graph.

we draw sign table

Vertical Sign Table for $y = f(x)$

function	$(-\infty, -5)$	$x = -5$	$(-5, 1)$	$x = 1$	$(1, 5)$	$x = 5$	$(5, \infty)$
$f(x)$.	$1/9 x^2 - 1/3 x + 2$	6	$1 - x$	0	$x - 1$	4	$\frac{1}{9}x^2 - \frac{1}{3}x + 2$
$f'(x)$.	$2/9 x - 1/3 < 0$	Undefined (corner)	$-1 < 0$	Transition.	$+1 > 0$	Undefined (corner)	$\frac{2}{9}x - \frac{1}{3} > 0$
$f''(x)$	$2/9 > 0$ (Concave up),	Transition	0 (linear)	0	0 linear	Transition	$2/9 > 0$ (concave up)
behaviour,	Decreasing Concave up.	Corner.	Decreasing Linear,	Local min	increasing linear.	corner.	Increasing Concave up.

(g) graph. :


f(x)
0.0
2.5
5
7.5
10.0
12.5
15.0
X
-10.0
-7.5
-5.0
-2.5
0.0
2.5
5.0
7.5
10.0
decreasing concave up.
x < -5
-5 (corner)
corner at x = -5 : (-5,6)
slope = -1 from (-5,6) to (1,0)
slope = +1 from (1,0) to (5,4)
5 corner
corner at x = 5 (5,4)
increasing concave up.
x > 5
1 (local min): (1,0)

- (For D and HD levels) The function

$$f(x) = \begin{cases} \sqrt{x^2 - 2x + 1} & |x| \leq c \\ ax^2 + bx + 2 & |x| > c \end{cases}$$

and for any $c \in N$.

(D1) find a and b in terms of c so that the function is continious in whole R.

Solution:-

we are tasked with finding coefficient a and b (in terms of c) such that the piecewise function

$$f(x) = \begin{cases} |x-1|, & |x| \leq c \\ ax^2 + bx + 2 & |x| > c \end{cases}$$

is continuous every where.

Step 1: Simplify the square root expression:-

$$\sqrt{x^2 - 2x + 1} = \sqrt{(x-1)^2} = |x-1|$$

Step 2: Continuity Conditions at Boundaries ($x = c$ and $x = -c$)

for continuity, the left-hand limit (LHL), right hand limit (RHL), and function value must match at the boundaries.

1. Continuity at $x = c$
   - LHL ($x \to c^-$):
     $$f(c^-) = |c-1|$$
   - RHL ($x \to c^+$):
     $$f(c^+) = ac^2 + bc + 2$$

Condition

$$\boxed{|c-1| = ac^2 + bc + 2 \text{ (Equation 1)}}$$

2. Continuity at $x = -c$
   - RHL ($x \to -c^+$)
     $$f(-c^+) = |-c-1| = c+1 \text{ (since } c \in N \Rightarrow c \geq 1)$$
   - LHL ($x \to -c^-$)
     $$f(-c^-) = a(-c)^2 + b(-c) + 2$$
     $$= ac^2 + (-bc) + 2$$
     $$= \cancel{ac^2 +} \; ac^2 - bc + 2$$

Condition:

$$\boxed{c+1 = ac^2 - bc + 2 \text{ (Equation 2)}}$$

Step 3: Solve the System of Equatins

we have two equations

1. $|c-1| = ac^2 + bc + 2$
2. $|c+1| = ac^2 - bc + 2$

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let's consider two cases based on value of c

Case 1: $c \geq 1$

In this case: $|c-1| = c-1$ & $|c+1| = c+1$

So equation becomes

1. $c-1 = ac^2 + bc + 2$
2. $c+1 = ac^2 - bc + 2$

Subtract the second equation from the first:

$$(c-1) - (c+1) = (ac^2 + bc + 2) - (ac^2 - bc + 2)$$
$$-2 = 2bc \Rightarrow b = -1/c$$

Now substitute b back into the first equation:

$$c-1 = ac^2 - 1/c \cdot c + 2$$

$$c-1 = ac^2 - 1 + 2 \Rightarrow c-1 = ac^2 + 1$$
$$ac^2 = c-2 \Rightarrow a = \frac{c-2}{c^2}$$

Case 2: $0 < c < 1$

In this case
$$|c-1| = 1-c \quad \text{and} \quad |c+1| = c+1$$

So the equation becomes

1. $1-c = ac^2 + bc + 2$
2. $c+1 = ac^2 - bc + 2$

Subtracting the second equation from the first:

$$(1-c)-(c+1) = (ac^2+bc+2)-(ac^2-bc+2)$$

$$-2c = 2bc \Rightarrow b = -1$$

Now substitube b back into the first equation:

$$1-c = ac^2-c+2 \Rightarrow 1 = ac^2+2$$

$$ac^2 = -1 \Rightarrow a = -1/c^2$$

however, for $c \in N$, $c \geq 1$, so this case does not apply here.

Final Answer:-

for $c \in N$ (i.e $c \geq 1$):

$$a = \frac{c-2}{c^2}, \quad b = -1/c$$

$$\boxed{a = \frac{c-2}{c^2}, \quad b = -1/c}$$

(D2) Is it possible to find a, b, c so that the function is differentiable in whole $\mathbb{R}$?

Solution: To determine whether its possible to find a, b and c such that the function is differentiable on all of $\mathbb{R}$, we must check:

1. Continuity at $x = c$ and $x = -c$ (already done in part D1) from previous solution.
2. Differentiability at $x = c$ and $x = -c$ - left-hand and right-hand derivatives must match at these points.

Recap of the function (after simplified).

$$f(x) = \begin{cases} \sqrt{x^2 - 2x + 1} & |x| \leq c \\ ax^2 + bx + 2 & |x| > c \end{cases}$$

After simplifying we get

$$f(x) = \begin{cases} |x - 1| & |x| \leq c \\ ax^2 + bx + 2 & |x| > c. \end{cases}$$

Step 1: Derivative of $f(x)$ for $|x| \leq c$

Since $f(x) = |x-1|$ its not differentiable at $x = 1$ (a corner point). So for differentiability at $x = c$ and at $x = -c$, the function must:

- be defined and continuous there (already ensured in D1)

$$\boxed{a = \frac{c-2}{c^2}, \quad b = -1/c}$$

- and the left-hand derivative = right-hand derivative at both points.

So check whether the derivative of $f(x) = |x-1|$ exists at $x = c$ and $x = -c$.

Case i: $x = c$

we know:

$$f(x) = \begin{cases} x-1 & x \geq 1 \\ -(x-1), & x < 1 \end{cases} \Rightarrow f'(x) = \begin{cases} 1 & x > 1 \\ -1 & x < 1 \end{cases}$$

So:

- If $c < 1$, then at $x = c$, $f(x) = -(x-1) \Rightarrow f'(x) = -1$
- If $c > 1$, then $f(x) = x-1 \Rightarrow f'(x) = 1$
- If $c = 1$, $f(x) = |x-1|$ is not differentiable at 1

So.

- only when $c \neq 1$, the derivative exist at $x = c$ within the $|x| \leq c$ domain

Now, derivative from both sides.

from $f(x) = ax^2 + bx + 2$ for $|x| > c$, derivative is:

$$f'(x) = 2ax + b$$

At $x = c$, set the left & right derivatives equal:

$$\text{if } c > 1 : \quad 1 = 2ac + b \quad \text{——— (1)}$$

$$\text{if } c < 1 : \quad -1 = 2ac + b \quad \text{——— (2)}$$

Case 2: $x = -c$

from earlier, $f(x) = |x-1|$

at $x = -c$, since $-c < 1$, $f(x) = -(x-1) \Rightarrow f'(x) = -1$

match with derivative of the polynomial side

$f'(-c) = 2a(-c) + b = -2ac + b.$ (3)

So for differentiability at $x = -c$:

$$\boxed{-2ac + b = -1} \quad (3)$$

Use continuity results from before:

we had: $a = \frac{c-2}{c^2}$, $b = -\frac{1}{c}$.

Now check if these values can satisfy the differentiability equation (1) (3)

Plug into equation (1) (assuming $c > 1$):

$$1 = 2ac + b = 2 \cdot \frac{c-2}{c^2} \cdot c + \left(-1/c\right)$$

$$= 2 \cdot \frac{c-2}{c} - 1/c$$

$$= \frac{2(c-2) - 1}{c}$$

$$= \frac{2c - 4 - 1}{c} = \frac{2c-5}{c}$$

Set equal to 1;

$$\frac{2c-5}{c} = 1 \Rightarrow 2c-5 = c \Rightarrow c = 5$$

Now try $c = 5$

· $a = \frac{5-2}{25} = 3/25$ · $b = \frac{-1}{5} = -1/5$

check equation (3)

$$-2ac + b = -2 \cdot 3/25 \cdot 5 - 1/5 = -\frac{30}{25} - 1/5 = -6/5 - 1/5$$

$$= -7/5 \neq -1$$

So it fails. Try to solve both equations symbolically now:

from (1):

$$2ac + b = 1 \quad (1)$$

from (3):

$$-2ac + b = -1 \quad (3)$$

Subtract (3) from (1):

$$(2ac+b) - (-2ac+b) = 1 - (-1) \Rightarrow 4ac = 2$$

$$ac = 1/2 \quad (4)$$

from continuity earlier;

$$a = \frac{c-2}{c^2} \Rightarrow ac = \frac{c-2}{c}$$

Set equal to (4)

$$\frac{c-2}{c} = 1/2 \Rightarrow 2(c-2) = c \Rightarrow 2c-4 = c \Rightarrow c = 4$$

Now try $c = 4$

- $a = \frac{4-2}{16} = 1/8$
- $b = -1/4 = -1/4$

Now check all:
Continuity at $x = c$:

$$ac^2 + bc + 2 = 1/8 \cdot 16 + (-1/4) \cdot 4 + 2 = 2 - 1 + 2 = 3$$

$$f(c) = |4-1| = 3 \Rightarrow \text{ok.}$$

Differentiability at $x = c$:

$f'(x) = 1$ from left.

$$2ac + b = 2 \cdot 1/8 \cdot 4 - 1/4 = 1 - 1/4 = 3/4 \neq 1$$

Still not equal → fails.

Final conclusion.

- The function involves an absolute value, which cause a sharp corner at $x = 1$.

So: If $c = 1$, the transition point is exactly at the non-differentiable point $x = 1$, and differentiability is not possible.

- if $c \neq 1$, then differentiability might be forced by choosing parameters - but equations do not simultaneously satisfy continuity and differentiability conditions

final Answer :

No, it is Not possible to find a, b, c such that the function is differentiable on all $\mathbb{R}$

(D3) for what values of c the function is differentiable at $x=c$ or $x=-c$?

Solution: To determine for which value of c the function $f(x)$ is differentiable at $x=c$ or $x-c$, we analyze the differentiability conditions at these points

Step 1: Simplifying the function

The function is given by:

$$f(x) = \begin{cases} \sqrt{x^2-2x+1} & \text{if } |x| \leq c \\ ax^2+bx+2 & \text{if } |x| > c \end{cases}$$

Notice that:

$$\sqrt{x^2-2x+1} = |x-1|.$$

Thus the function can be rewritten as

$$f(x) = \begin{cases} |x-1| & \text{if } |x| \leq c \\ ax^2+bx+2 & \text{if } |x| > c. \end{cases}$$

Step 2: Continuity Condition

from D1, we have the continuity conditions at $x=c$ and $x=-c$

$$|c-1| = ac^2+bc+2, \quad |-c-1| = ac^2-bc+2$$

for $c \geq 1$, these simplify to:

$$c-1 = ac^2+bc+2 \quad , c+1 = ac^2-bc+2.$$

Solving these gives.

$$\boxed{a = \frac{c-2}{c^2}, \quad b = -1/c.}$$

Step: 3 Differentiability at $x = c$

for $f(x)$ to be differentiable at $x = c$, the left-hand and right-hand derivatives must be equal.

- left hand derivative (for $|x| \le c$):

$$f_1'(x) = \begin{cases} 1 & \text{if } x > 1 \\ -1 & \text{if } x < 1 \end{cases}$$

from $f(x) = |x-1|$
if $c < 1$
$f'(c) = -1$

at $x = c \geq 1$, $f_1'(c) = 1$.

if $c > 1$
$f'(c) = 1$

- Right-hand derivative (for $|x| > c$):

$$f_2'(x) = 2ax + b.$$

at $x = c$, $f_2'(c) = 2ac + b$.

Setting them equal:

$$1 = 2ac + b.$$

Substituting $a = \frac{c-2}{c^2}$ $b = -1/c$

$$1 = 2\left(\frac{c-2}{a^2}\right)c - 1/c = \frac{2c(c-2)}{c} - 1/c$$

$$= \frac{2c-4-1}{c} = \frac{2c-5}{c}$$

Solving

$$1 = \frac{2c-5}{c} \Rightarrow c = 2c-5 \boxed{\Rightarrow c = 5}$$

Step 4: Differentiability at $x = -c$

Similarly, for differentiability at $x = -c$.

- left-hand derivati (for $|x| \leq c$):
  At $x = -c$, since $-c < 1$, $f'_1(-c) = -1$
- Right-hand derivative (for $|x| > c$):
  At $x = -c$, $f_2'(-c) = -2ac + b$

Setting them equal:

$$-1 = -2ac + b$$

Substituting

$$a = \frac{c-2}{c^2} \text{ and } b = -1/c :$$

$$-1 = -2\left(\frac{c-2}{c^2}\right)c - 1/c = \frac{2c(-2)}{c} - 1/c$$

$$= \frac{-2c+4-1}{c} = \frac{-2c+3}{c}$$

Solving:

$$-1 = \frac{-2c+3}{c} \Rightarrow -c = -2c+3 \boxed{\Rightarrow c = 3}$$

Step 5: Verification

for $c = 5$

Differentiable at $x = 5$ but not necessarily at $x = -5$.

for $c = 3$

Differentiable at $x = -3$ but not necessarily at $x = 3$

However, we must also the function is differentiable at $x = 1$ (where $|x-1|$ has a sharp corner). for $c \geq 1$, $x = 1$ lies within $|x| \leq c$, making $f(x)$ non-differentiable at $x = 1$ unless the quadratic piece matches the derivative there, which is imposible due to the corner.

Final Answer

The function $f(x)$ is differentiable at $x = c$ only when $c = 5$ and differentiable at $x = -c$ Only when $c = 3$, However, due to the inherent non-differentiability at $x = 1$, the function cannot be differentiable everywhere in $\mathbb{R}$ for any c.

The function is differentiable at $x = c$ only when $c = 5$ & differentiable at $x = -c$ only when $c = 3$

<---------- End ---------->